A NUMERICAL TECHNIQUE FOR THE CALCULATION OF CLOUD OPTICAL EXTINCTION FROM LIDAR

J.M. Alvarez, LaRC, MS 417, Hampton, VA 23681 (804) 864-2677
M.A. Vaughan, SAIC, MS 417, Hampton, VA 23681 (804) 864-5331

ABSTRACT

A simple numerical algorithm which calculates optical extinction from cloud lidar data is presented. The method assumes a two-component atmosphere consisting of "clear air" and cloud particulates. "Clear air" may consist of either molecules only or a mix of molecules and atmospheric aerosols. For certain clouds, the method may be utilized to provide an estimate of the cloud-atmospheric parameter defined as the ratio of the cloud volume backscatter coefficient to the cloud extinction coefficient divided by the atmospheric volume backscatter coeffficient at a given altitude. The cloud-atmospheric parameter may be estimated only from cloud data from which the optical thickness may reliably be used as a constraint on the numerical solution. This constraint provides the additional information necessary to obtain the cloud-atmospheric parameter. Conversely, the method may be applied to obtain cloud extinction and optical thickness from lidar cloud soundings if an estimate of the cloud-atmospheric parameter is available.

INTRODUCTION

The equation(Liou 1980) underlying all attempts to obtain optical extinction from lidar data is given by

$$E(r) = \frac{c\tau\beta A_r}{8\pi r^2} e^{\left(-2\int_0^r \sigma(r')\,dr'\right)} P \tag{1}$$

where E is the backscattered power received by a lidar system emitting power P equipped with a telescope of effective area A_r, c is the speed of light, τ is the laser pulse length, ß is the volume backscattering coefficient, r is the range, and σ is the extinction coefficient. Published solutions to this equation are so numerous and well developed that only very few are cited(Viezee 1969, Collis 1976, Barrett and Ben-Dov 1967, Davis 1969, Klett 1981). If atmospheric scattering and cloud aerosol scattering are to be explicitly considered, then the lidar equation contains two components and is given by

$$E(r) = D\frac{\beta_a+\beta_c}{r^2} e^{-2\int_0^r [\sigma_a(r')+\sigma_c(r')]\,dr'} \tag{2}$$

where D is $c\tau A_r P/8\pi$ and the a and c subscripts refer to the atmospheric(air) and the cloud parts of the signal. Solutions for equation 2 are not as well developed as for equation 1 partly because there is no analytic solution for this equation as it stands. At present only iterative solutions for equation 2 have been proposed(Fernald 1972, Fernald 1983, Klett 1985).

We present a simple numerical method for the calculation of optical extinction from the two component lidar equation . This new method modifies equation 2 so that a straightforward calculation yields optical extinction.

TECHNIQUE

We start by noting that the numerical array comprising each lidar sounding is proportional to equation 2. *We assume that a "clear" atmosphere is a combination of molecules and the background atmospheric aerosol.* At altitudes above and below the cloud, equation 2 holds but $\beta_c = \sigma_c = 0$. If equation 2 is divided by the lidar signal from a "clear" atmosphere as determined from a lidar sounding in a clear region below the cloud or from an atmospheric model fit to the clear air part of the lidar return, the attenuated scattering ratio, R_{sc}, is obtained. In the cloud, this ratio is given by

$$R(r)_{sc} = (1+\frac{\beta_c}{\beta_a})\, e^{-2\int_{r_b}^{r}\sigma_c(r')_c dr'} \tag{3}$$

where r_b is cloud base altitude, and the subscripts "a" and "c" refer to "clear air" and cloud respectively. This ratio is obviously 1 below the cloud. In the region above the cloud the attenuated scattering ratio is given by

$$R_{sc} = e^{-2\int_{r_b}^{r_t}\sigma_c(r')_c dr'} \tag{4}$$

where r_t is cloud top altitude. This equation implies that in certain cases, the cloud optical thickness can be obtained directly from the lidar data via the attenuated scattering ratio .

We next assume that the atmospheric volume-backscattering coefficient is proportional to the atmospheric molecular number density N(r) for altitudes greater than some minimum altitude. This assumption is supported by the aerosol data presented by Wallace(1977) who showed that averages of aerosol number densities obtained from Aiken particle counters exhibited the same altitude distribution as the molecular number density at altitudes above about 3 or 4 kilometers. If the molecular number density for the higher altitudes is given by

$$N(r) = N_0\rho(r) \tag{5}$$

where N_0 is the number density at altitude r_0 and ρ contains the altitude dependence of the molecular number density for altitudes greater than r_0, then the atmospheric volume backscatter coefficient $\beta(r)$ is given by

$$\beta(r) = \beta_0\rho(r) \tag{6}$$

where β_0 is the atmospheric volume backscatter coefficient at altitude r_0. If we further assume that, for the cloud, the ratio of backscatter volume to extinction coefficients is a constant, k_c(equal to β_c/σ_c) then, equation 3 may be rewritten as

$$R_{sc} = (1+\frac{k_c}{\beta_0}\frac{\sigma_c}{\rho(r)})\, e^{-2\int_{r_0}^{r}\sigma_c(r')dr'} \tag{7}$$

Note that R_{sc} is a function of the cloud-atmospheric parameter(CAP), k_c/β_0 and the functions $\sigma_c(r)$ and $\rho(r)$. Since the molecular number density altitude distribution can be obtained from radiosonde data or a model, $\rho(r)$ can be considered known.

We will initially assume that the CAP(the ratio k_c/β) is known; later we will see how this ratio can itself be determined from lidar data. If we suppose that the CAP is known, then equation 7 contains only one "unknown", the extinction $\sigma_c(r)$. We translate equation 7 to a numerical equation by subdividing the cloud into layers of equal thickness in altitude. We then use the trapezoid rule to obtain

$$R_i = (1+\frac{k_c}{\beta_0}\frac{\sigma_i}{\rho_i})\, e^{(-\sigma_0\Delta - 2\Delta(\sigma_1+\sigma_2+\ldots\sigma_{i-1}) - \sigma_i\Delta)} \tag{8}$$

where the "i" corresponds to different altitudes with i=0 being the first cloud layer(stratum), and i=K being cloud top; the finite differential altitude is Δ. The one-half weighting on the first and last elements in the sum in the exponential is in accordance with the trapezoid rule.

We start our calculation at cloud bottom(i=0) where only the first extinction term, σ_0, is present. At this first point, equation 8 is a transcendental equation in σ_0 and we solve this equation numerically by Newton's method. We then consider the second point(i=1) and since we know R_1, ρ_1, σ_0, and the CAP, we again obtain σ_1 numerically. We continue in this manner until we obtain the entire extinction array. At the end of this process we sum the extinction coefficients and calculate the cloud optical thickness, thus completing the calculation.

If the CAP is unknown, the problem is more challenging. To obtain a solution under these conditions requires cloud lidar returns having adequate atmospheric signal at altitudes above and below the cloud. Equation 4 can be utilized to measure the optical thickness for such clouds. We initially set the CAP to some arbitrary value. For this, the extinction coefficient array is calculated as before and the optical thickness is estimated by summing the extinction. Then, this optical thickness estimate is compared to the lidar-measured optical thickness obtained by equation 4. If the two are not sufficiently close the CAP is changed and the entire solution process is repeated. This cycle is reiterated until the optical thickness from the inversion is substantially the same as the measured optical thickness. This analysis thus obtains the cloud optical extinction array and the CAP which can then be used to calculate the optical thickness of denser clouds.

EXAMPLE

Figure 1 below presents a simulated lidar signal function(the natural logarithm of the range-squared corrected lidar return) used to test the inversion algorithm. The lidar return is one which would be expected from a noiseless sounding of an exponential Rayleigh atmosphere having a scale height of 9 km. The atmospheric extinction for the simulated lidar return is 0.008/km at an altitude of 2 km; this extinction fixes the value of β_0 at 2 km. The cloud optical thickness is assumed to be 0.100. The extinction coefficient has the parabolic-shaped altitude distribution shown by the middle line(labeled (o) in figure 2). The same middle line in figure 2 corresponds to the inverted values of extinction if the optical thickness constraint used in the inversion is the correct one. The rest of the curves presented in figure 2 correspond to inversions having incorrect values of cloud optical thickness, Γ, thus resulting in incorrect values for the cloud extinction coefficients and the cloud backscatter/extinction ratio, k_c, used here in place of the CAP. In actual cloud lidar returns, the measured cloud optical thickness is obtained from noisy atmospheric signals above and below the cloud. The atmospheric signal noise results in errors in the inferred optical thickness. The errors made in the input data and the consequent incorrect values for the backscatter/extinction ratio, k_c, are presented in Table 1 following Figures 1 and 2.

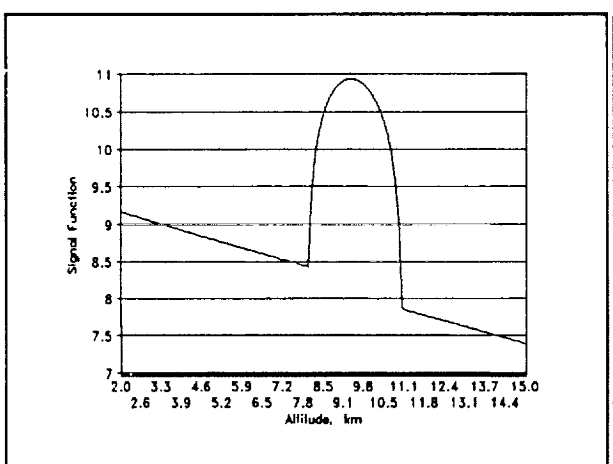

Figure 1. Simulated Lidar Return Signal Function.

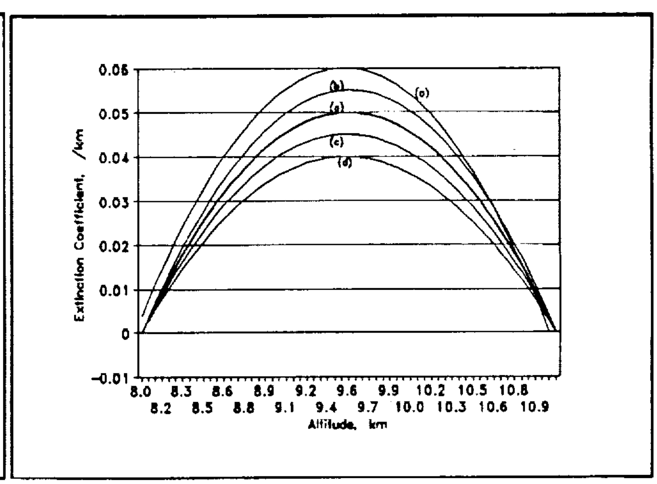

Figure 2. Simulated Return Extinction Coefficients.

Table I. Error Analysis Results

Per cent error in Γ	20.0%	10.0%	0	-10.0%	-20.0%
Per cent error in k_c	-15.2%	- 8.0%	0	9.6%	22.4%

Perusal of the two figures and the accompanying values given in Table 1 above indicate that if the optical thicknesss used to invert the k_c's are too high, the extinction coefficients are too high and the corresponding k_c's are too low and vice versa.

The accuracy of the inversion is also affected by other factors. One of these factors is the noise in the atmospheric lidar data below the cloud. The reason for this effect is that the attenuated scattering ratio in the cloud is obtained by fitting the actual lidar signal below the cloud to the expected atmospheric lidar signal. Thus, errors in the fit below the cloud produce errors in the attenuated scattering ratio which translate into errors in the CAP and the optical thickness. Another factor contributing to inversion errors is the noise present in the lidar data in the cloud itself. These errors likewise translate into errors in the final results. We are performing an extensive error analysis of the inversion. Thus far it appears that noise in the lidar data does not produce errors in the inversion which rapidly escalate or behave in an unstable fashion.

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